Algorithm for Gear Hub Tool Profiling by Bezier Polynomial Approximation. The Rotary Helical Screw Compressor Case.

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Abstract: - Gear hobs tool reciprocally enwrapping a profiles whirl, associated with a rolling centrodes are usually designed using the second Olivier theorem [5], enwrapped surfaces having punctiform contact. In the case of rotary helical screw an approximation method by 2nd or 3rd degree Bezier polynomials is proposed, simplifying calculus but still preserving profiling accuracy. Numerical examples are also presented, based on dedicated software.

Key-Words: - Gear hub, Bezier polynomial approximation, rotary helical screw.

1 Introduction

Already know are methods for gear hub primary peripheral surface profiling [1], [4], [5], [6], reciprocally enwrapping with an ordered surfaces whirl, with punctiform contact, based on fundamental theorems: second Olivier theorem, Grohman theorem, intermediary surface method or decomposition of helical movement – Nikolaev[5].

Complex profiles, as is the case of rotary helical screw compressors, can be modeled using analytical representation for those profiles, hear hob profiling being realized using already know theoretical methods.

Also, the problem could be solved, with satisfactory technical precision, by substituting analytical profiles of arcs and segments with parametric Bezier polynomials. The reduced number of points one should know to apply this methodology and also uniformity and mathematical simplicity can be considered important advantages for the proposed method.

2 Rack gear surface

We analyze further, see fig. 1, the rack profile, generating the rotary helical screw compressor. The rack profile is constituted by the following elementary profiles:

AB – circle arc with radius R_0;

BC – straight line segment;

CD – circle arc with radius r_0;

AH – Bezier polynomial curve arc;

HG – Bezier polynomial curve arc;

GF – straight line segment;

FE – circle arc with radius R_0;

Consecutive elementary profiles are tangent in the common point. Therefore, continues gear hob tools, generating rotary helical screw, are obtained.

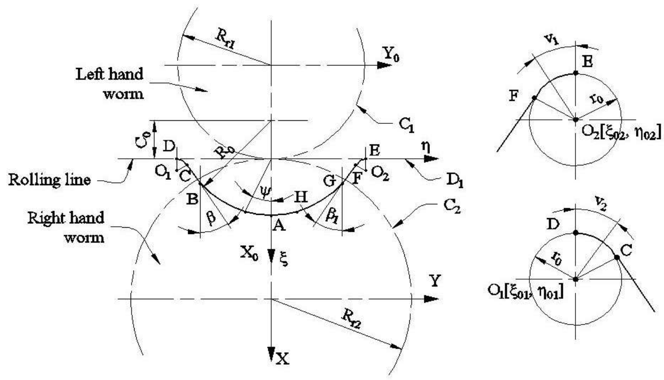

Fig.1 . Rack gear generating the rotary helical screw compressor

Let consider the parametric equations for the elementary profiles of the rack:

- cylindrical surfaceAB

$$S_{AB}\left|\begin{array}{l}\xi = R_0 \cos\psi - C_0;\\ \eta = -R_0 \sin\psi + t \sin\beta_d\\ \zeta = t\cos\beta_d.\end{array}\right. \qquad (1)$$

C_0 - is a constant, t and ψ are variable parameters; $0 \le \psi \le \psi_{\max}; \psi_{\max} = \frac{\pi}{2} - \beta$;

- planar surface BC

$$S_{BC}\left|\begin{array}{l}\xi=\xi_B-u\cos\beta;\\ \eta=\eta_B+u\sin\beta+t\sin\beta_d;\\ \zeta=t\cos\beta_d,\end{array}\right. \quad (2)$$

t,u - are variable parameter, $\beta=\frac{\pi}{2}-\psi_{\max}$; ξ_B,η_B - are determined from eq. (1) when $\psi=\psi_{\max}$

- cylindrical surface CD

$$S_{CD}\left|\begin{array}{l}\xi=\xi_{O_1}-r_0\cos v;\\ \eta=\eta_{O_1}+r_0\sin v+t\sin\beta_d;\\ \zeta=t\cos\beta_d,\end{array}\right. \quad (3)$$

t and v are variable parameters; ξ_{O_1},η_{O_1} - coordinates of the center of the circle are constructive values,

$$0\le v\le\frac{\pi}{2}-\beta; \quad (4)$$

- cylindrical surfaces AH and HG

For AH we consider a 2nd degree Bezier polynomial

$$S_{AH}\left|\begin{array}{l}\xi=P_{\xi_{AH}}=\lambda_1^2A_\xi+2(1-\lambda_1)\lambda_1B_\xi+\\ \quad+(1-\lambda_1)^2C_\xi;\\ \eta=P_{\eta_{AH}}=\lambda_1^2A_\eta+2(1-\lambda_1)\lambda_1B_\eta+\\ \quad+(1-\lambda_1)^2C_\eta+t\sin\beta_d;\\ \zeta=t\cos\beta_d.\end{array}\right. \quad (5)$$

where $0\le\lambda_1\le1$,

and similarly

$$S_{HG}\left|\begin{array}{l}\xi=P_{\xi_{HG}}=\lambda_1^2D_\xi+2(1-\lambda_1)\lambda_1E_\xi+\\ \quad+(1-\lambda_1)^2F_\xi;\\ \eta=P_{\eta_{HG}}=\lambda_1^2D_\eta+2(1-\lambda_1)\lambda_1E_\eta+\\ \quad+(1-\lambda_1)^2F_\eta+t\sin\beta_d;\\ \zeta=t\cos\beta_d.\end{array}\right. \quad (6)$$

where $0\le\lambda_2\le1$.

Polynomial coefficients above are determined by the following conditions (for any arbitrary fixed value of parameter t):

- common point - A, and common tangent - in A, between S_{AB} and S_{AH};
- common point – G, and common tangent – in G, between S_{FG} and S_{HG};
- common point – H, and common tangent – in H, between S_{AH} and S_{HG};

- planar surface:

$$S_{FG}\left|\begin{array}{l}\xi(u_1)=\xi_F+u_1\cos\beta_1;\\ \eta(u_1)=\eta_F-u_1\sin\beta_1+t\sin\beta_d;\\ \zeta=t\cdot\cos\beta_d,\end{array}\right. \quad (7)$$

where $0\le u_1\le u_{1_{\max}}$, and β_1 are constructive parameters;

- cylindrical surface:

$$S_{CD}\left|\begin{array}{l}\xi=\xi_{O_2}-r_0\cos v_1;\\ \eta=[L_P-\eta_{O_2}]+r_0\sin v_1+t\sin\beta_d;\\ \zeta=t\cos\beta_d,\end{array}\right. \quad (8)$$

where ξ_{O_2},η_{O_2} - coordinates of the center of the circle are constructive values and L_P is the length of the rack profile along the translation axis.

Rack profile obtained as above, in rolling motion, generates two helical surfaces of the rotary compressor (male and female). Moreover, once the rack profile is know, the cylindrical surface directed by curve assembly DCBAHGFE and the versor $\vec{t}$ can be constructed;

$$\vec{t}=\sin\beta\cdot\vec{j}+\cos\beta\cdot\vec{k}, \quad (9)$$

where β is the inclination angle of the helical screw:

$\tan\beta=\dfrac{P_E}{2\pi R_{r_2}}$ (where R_{r_2} is the worm radius)

3 Gear Hub Primary Peripheral Surface Profiling. Algorithm

The profile of the rack and positions of the coordinate's systems axis were defined. We have to determine the characteristics of rack cylindrical surface in contact with primary peripheral surface of the gear hob, see fig. 2.

The helical movement generating primary peripheral surface of the gear hob $(\vec{V},p)$, can be decomposed in a sum of equivalent movements:

- translation along the vector $\vec{t}$ - rack cylindrical surface generator;
- rotation around axis $\vec{A}$ (parallel to vector $\vec{V}$ – the helical surface axis – and at a distance $a=p\cdot\tan\theta$ from $\vec{V}$).

In the equations above, p denotes helical parameter of the gear hob, and θ is the angle between vectors $\vec{V}$ and $\vec{t}$, see fig. 2.

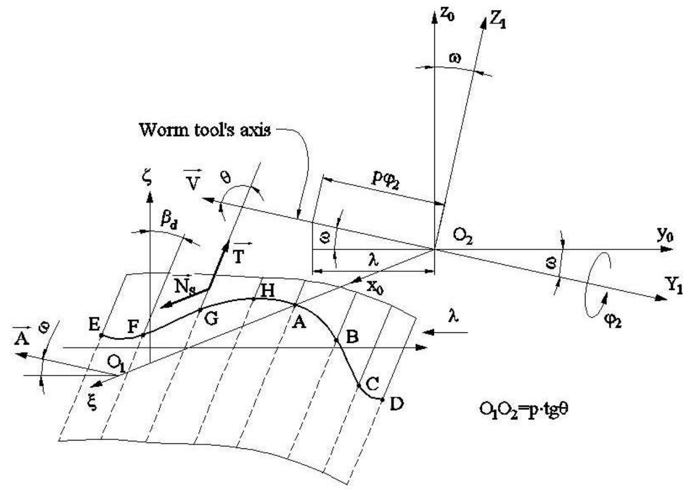

Fig.2. Decomposition of the helical movement

Since $\overrightarrow{N_S} \cdot \vec{t} = 0$, the characteristically curve for surface S (rack cylindrical surface) depends only the rotation movement and not the translation component of the movement. Characteristically curve is actually the projection on surface S of the axis $\vec{A}$.

Therefore, characteristically curve is the geometric place of those points of S such that $\overrightarrow{N_S}$ intersects $\vec{A}$.

Let:

- axis $\vec{A}$, in $x_0 y_0 z_0$ - fixed reference system associated to gear hob primary peripheral surface, be:
$$\vec{A} = -\cos\omega \cdot \vec{j} + \sin\omega \cdot \vec{k}; \qquad (10)$$
- normal vectors at cylindrical surface S, be:
$$\overrightarrow{N_S} = N_{x_o} \cdot \vec{i} + N_{y_o} \cdot \vec{j} + N_{z_o} \cdot \vec{k}, \qquad (11)$$

see table 1; (coordinates system $\xi\eta\zeta$ has parallel axes and same orientation as $x_0 y_0 z_0$)

- current vector be
$$\vec{r_1} = \overline{O_2 O} \cdot \vec{i} + \vec{r}, \qquad (12)$$

where $\vec{r}$ is the vector indicating current point on the cylindrical surface S and $\overline{O_2 O} = R_{r_S}$ - gear hob radius – a technological parameter.

A similar solution for the same problem can be obtained by replacing profiles in equations (1-3, 6-7) with low degree Bezier polynomials imposing the condition that adjacent profiles have continuous tangent.

The value of the parameter ω is determined from the condition that the unfolded helical line of radius R_{r_s} is parallel to vector $\vec{t}$.

$$\omega = \beta_d \pm \omega_s, \tan\omega_s = \frac{p}{R_{r_s}}. \qquad (13)$$

Condition used to determine characteristically curve, in the case of helical movement decomposition, has the following form:

$$(\vec{A}, \overrightarrow{N_s}, \vec{r_1}) = 0, \qquad (14)$$

or, in extended form:

$$\begin{vmatrix} N_{X_0} & N_{Y_0} & N_{Z_0} \\ X_0 - a & Y_0 & Z_0 \\ 0 & -\cos\omega & \sin\omega \end{vmatrix} = 0, \qquad (15)$$

where X_0, Y_0, Z_0 are obtained from equations (1-7) using the transformation:

$$\begin{vmatrix} X_0 \\ Y_0 \\ Z_0 \end{vmatrix} = \begin{vmatrix} \xi \\ \eta \\ \zeta \end{vmatrix} - \begin{vmatrix} -R_r \\ 0 \\ 0 \end{vmatrix}. \qquad (16)$$

Table 1. Normal vectors for each portions of the rack cylindrical surface

Portion	Normal vector
S_{AB} (see 1)	$\overrightarrow{N_{S_{AB}}} = (-\cos\psi\vec{i} + \sin\psi\vec{j})\cos\beta_d - -\sin\psi\sin\beta_d\vec{k}$
S_{BC} (see 2)	$\overrightarrow{N_{S_{BC}}} = (\sin\beta\vec{i} + \cos\beta\vec{j})\cos\beta_d - -\cos\beta\sin\beta_d\vec{k}$
S_{CD} (see 3)	$\overrightarrow{N_{S_{CD}}} = (\cos v\vec{i} - \sin v\vec{j})\cos\beta_d + +\sin v\sin\beta_d\vec{k}$
S_{AH} (see 4)	$\overrightarrow{N_{S_{AH}}} = (\frac{\partial\eta}{\partial\lambda_1}\vec{i} - \frac{\partial\xi}{\partial\lambda_1}\vec{j})\cos\beta_d + +\frac{\partial\xi}{\partial\lambda_1}\sin\beta_d\vec{k}$
S_{HG} (see 5)	$\overrightarrow{N_{S_{HG}}} = (\frac{\partial\eta}{\partial\lambda_2}\vec{i} - \frac{\partial\xi}{\partial\lambda_2}\vec{j})\cos\beta_d + +\frac{\partial\xi}{\partial\lambda_2}\sin\beta_d\vec{k}$
S_{FG} (see 6)	$\overrightarrow{N_{S_{FG}}} = (-\sin\beta_1\vec{i} - \cos\beta_1\vec{j})\cos\beta_d + +\cos\beta_1\sin\beta_d\vec{k}$
S_{EF} (see 7)	$\overrightarrow{N_{S_{EF}}} = (\cos v_1\vec{i} - \sin v_1\vec{j})\cos\beta_d + +\sin v_1\sin\beta_d\vec{k}$

Equations (1-7) and condition (15) determine the characteristically curve for each portion of the rack cylindrical surface, denoted by:

$C_s : [C_{S_{AB}}, C_{S_{BC}}, C_{S_{CD}}, C_{S_{AH}}, C_{S_{HG}}, C_{S_{FG}}, C_{S_{EF}}]$

This set of curves represents the tangency between surface S and gear hub primary peripheral surface.

In helical movement around the axis $\vec{V}$ with helical parameter p, the curve C_S generates gear hub primary peripheral surface:

$$\begin{Vmatrix} X_1 \\ Y_1 \\ Z_1 \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{Vmatrix} \cdot \begin{Vmatrix} X_{C_S} \\ Y_{C_S} \\ Z_{C_S} \end{Vmatrix} + \begin{Vmatrix} 0 \\ p\varphi \\ 0 \end{Vmatrix} \tag{17}$$

Gear hub primary peripheral surface is a parametric surface:

$$\Pi : \begin{vmatrix} X_1 = X_1(\lambda_i, \varphi); \\ Y_1 = Y_1(\lambda_i, \varphi); \\ Z_1 = Z_1(\lambda_i, \varphi), \end{vmatrix}$$

where λ_i are generic variables for equations (1-7).

Imposing the condition

$$Z_1 = 0 \tag{18}$$

the axial section of the gear hob is obtained.

Condition (18) represents a relation between λ_1 and φ, therefore the action section has a generic form:

$$\Pi_A : \begin{vmatrix} X_1 = X_1(\lambda_i); \\ Y_1 = Y_1(\lambda_i). \end{vmatrix}$$

4 Numerical examples

We present further an example of a female rotary helical screw compressor having the following constructive parameters: he distance between the axes of both rotary screws, $A_{12} = 80mm$; $r_0 = 1.1mm$; $R_0 = 22mm$.

Control points, from equations (1-7), in YX coordinates system are:

D: [-25.267; -52.000]; C: [-24.282; -51.357];
B: [-19.671;-42.149]; A: [6.938; -30.000];
G: [20.671; -45.957]; E: [24.057; -51.444];
F: [24.998; -52.000].

The following table represents several key points of the gear hob axial section.

Table 3. Axial section of the gear hob

X	Y
46.0000	25.5901
46.1116	25.1343
⋮	⋮
49.4118	22.5863
46.6735	24.3122
46.1423	24.7947
46.0000	25.3182

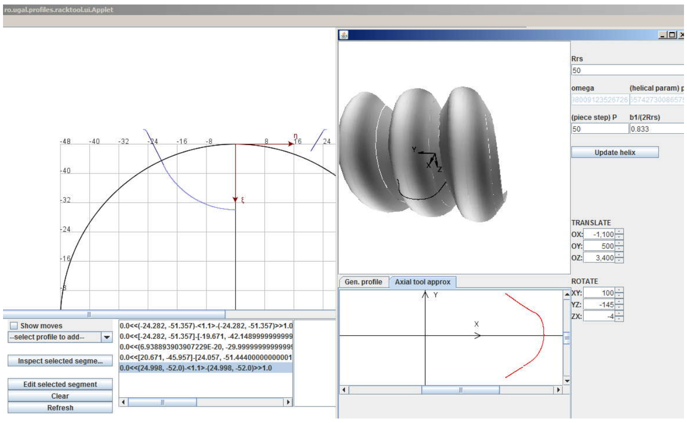

Fig.3 . Software application. Rack profile, gear hob, characteristically curve and axial section

4 Conclusion

The profiling method of the gear hub which generates an ordered profiles whirl is based on the principles of the helical motion decomposition.

The method use the Bezier approximation polynomials for the rack-gear reciprocally enwrapping with profiles whirl and is rigorous enough, for usually profiles

A special software, written in Java programming language, was specially design for computing polynomial approximation and the axial section of the gear hob.

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